

# Microscopic picture of superfluid $^4\text{He}$

Yongle Yu  $\dagger$  and Hailin Luo  $\ddagger$

$\dagger$  State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Science, West No. 30 Xiao Hong Shan, Wuchang, Wuhan, 430071, China

$\ddagger$  Fujian Institute of Research on the Structure of Matter, Chinese Academy of Sciences, 155 Yangqiao Road West, Fuzhou, 350002, China

**Abstract.** We illustrate the microscopic quantum picture of superfluid  $^4\text{He}$  with the help of revealing a hidden property of its many-body levels. We show that, below the transition point, the low-lying levels of the system form a grouping structure with each level belonging to one specific group only. In a superflow state or a static state, the system establishes a group-specific thermal equilibrium with its environment and the levels of an initially-occupied group shall be thermally distributed. The other initially-unoccupied groups of levels remain unoccupied, due to the fact that inter-group transitions are prohibited. The macroscopically observable physical quantities of the system, such as superflow velocity and thermal energy density, are determined statistically by the thermal distribution of the occupied group(s). We further show that thermal energy of a superflow has an unusual flow velocity dependence: the larger the velocity is, the smaller the thermal energy. This velocity dependence is responsible for several intriguing phenomena of the system, such as the mechano-caloric effect and the fountain effect, which demonstrate a fundamental coupling between the thermal motion of the system and its hydrodynamic motion. We report an experimental observation of a counter-intuitive self-heating effect of  $^4\text{He}$  superflows, which confirms that a  $^4\text{He}$  superflow carries significant thermal energy depending on its velocity.

## I. Introduction

The discovery of quantum mechanics is a paradigm shift event in the evolution of the modern civilization. Not only it advances our knowledge of physical systems to a fundamentally deep level, but also it leads to many important technologies that pervade every aspect of life. Despite its many counter-intuitive features (such as quantum entanglement and tunneling), quantum mechanics provides an unified framework to describe various physical systems in a lab, such as atomic and molecular systems, condensed matter systems, .... This quantum framework is fundamental and mathematical-physically intrinsic. It formulates that the physical states of any system

can be described by a set of quantum wavefunctions (states) of the system, and that the physical processes of the system correspond to the transitions among the quantum states.

In the field of atomic and molecular physics, many high accurate agreements between the quantum theory and the experimental observations have been achieved. These impressive accomplishments are partially attributed to the fact that since the number of particles in some atomic and molecular systems is limited, accurate calculations of the quantum properties of the systems can be performed.

In the field of condensed matter physics, the quantitative agreements, and sometimes even the qualitative agreements between the quantum theory and the experimental observations, are far less satisfied. Reliable calculations of quantum properties of a condensed matter system are often technically intractable since one is dealing with a large number of particles in the system (i.e., the quantum many-body problem). Theoretical approximations are made to deal with this problem. One of the most frequently used approaches is the mean field or single particle approach, where the quantum state of the system is approximated by a product of single-particle quantum wavefunctions. Many important phenomena can be understood in this mean field framework, such as the differences between conductors, semiconductors and insulators. However, this approach turns out to be insufficient for a number of intriguing quantum phenomena. There are some exceptions beyond the mean field approach, for example, the Laughlin wavefunctions for the description of the fractional quantum Hall effect.

One of the most fundamental condensed matter systems is superfluid  $^4\text{He}$  which demonstrates a wide variety of unusual quantum behaviors. The primary dissipationless phenomenon of superfluidity [1, 2] is somehow unthinkable from the general viewpoint of the second law of thermodynamics, which typically requires that a closed system shall reach a state of maximum entropy by dissipating all energy of an ordered motion into thermal energy. Several other phenomena of superfluid  $^4\text{He}$ , such as the fountain effect [3] the mechano-caloric effect [4], reveal a fundamentally intriguing coupling between the hydrodynamic motion of the system and its thermal motion. A naturally posed question is whether one can have a clear quantum understanding of these unusual behaviors in the direct terms of quantum states and quantum transitions among the state. Can one develop a quantum picture of superfluid  $^4\text{He}$  which is as straightforward as the quantum picture of an atomic system? In this paper, we give a yes answer to the question. We show that with some general physical arguments, the low-lying many-body levels of the system are actually grouped, with each level belonging to one specific group only. At low temperature the system can establish a group-specific thermal equilibrium with its surroundings and it can support a macroscopic superflow state depending on the occupied group. We further show that this grouping structure of many-body levels leads naturally to another unusual property of the system. The thermal energy of a superflow has a group-dependence which can be transformed into a velocity-dependent behavior: the larger the velocity, the smaller the thermal energy. This velocity dependence is responsible for the fundamental coupling between the system's hydrodynamic motion

and its thermal motion.

In the literatures there are a large number of important theoretical works which contribute significant understandings to various aspects of superfluid  $^4\text{He}$  (see, for example, [5, 6, 7, 8, 9, 10, 11, 12, 13] and the references in [14]). The theoretical development in this paper would be impossible without the inspirations of some of these previous works.

The rest of paper is organized as follows. Section II argues that the two-fluid model of superfluid  $^4\text{He}$  has a fundamental flaw and mentions several experimental observations which contradict the two-fluid model. The problems of this phenomenological model strengthen the necessity of pursuing the direct microscopical picture of the system. Section III illustrates some general properties of the quantum states of superfluid  $^4\text{He}$  and emphasizes on the grouping structure of the low-lying levels of the system. Section IV argues that the thermal energy of superfluid  $^4\text{He}$  has a natural dependence on the flow velocity, based on quantum statistics of the occupied group(s) of microscopic levels and the properties of Galilean transformation. Section V reports an intriguing self-heating phenomenon of  $^4\text{He}$  superflows, which confirms that superflow  $^4\text{He}$  can carry significant thermal energy depending on the flow velocity. The conclusions are drawn in Section VI.

## II. Fundamental flaw of the two-fluid model and its contradictions with experimental observations.

The two-fluid model of superfluid  $^4\text{He}$  plays an essential role in the understandings of many behaviors of the system. However, this phenomenological model has a fundamental flaw which challenges its reliability in a substantial way.

In the two-fluid model, a superfluid component is proposed and characterized by a very exotic property: it has zero entropy. Following this assumption, one shall not be prevented from investigating another important thermal physical quantity of this component, which is its temperature. Then it must have a temperature of absolute zero in order to be consistent with zero entropy. The question is how this component at absolute zero can coexist with its thermal surroundings. Such a co-existence is more 'miraculous' than finding a piece of ice in a steel melt. The zeroth law of thermodynamics rules out this co-existence in an absolute way. Later we shall show that the microscopic mechanism of superfluid  $^4\text{He}$  is free of any unphysical zero-entropy assumption and is consistent with zeroth law of thermodynamics.

In the two-fluid model of superfluid  $^4\text{He}$ , the treatment of the normal density and the superfluid density requires that the sum of these two densities remains constant which corresponds to the density of all helium atoms. The normal density is computed as the density of quasiparticles (phonons and roton) and one quasiparticle corresponds exactly to one helium atom in the total density. After excluding the quasiparticle-mapped atoms, the rest of all helium atoms in the system forms the superfluid component. The concept of two components is clearly a bipartition of all helium atoms in the

system. However, this bipartition is unbacked at the fundamental level. By the principle of quantum mechanics, the microscopic state of the system is specified by a large number of many-body wavefunctions  $\psi_\alpha(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  (which be conveniently taken as the eigenstates of the Hamiltonian operator of the system,  $\alpha$  labels the eigen levels and  $N$  is the number of atoms). The integrity of each wavefunction demands that all the atoms should be treated as an indivisible wholeness, and each wavefunction is embedded with complex intrinsic quantum correlations among all the atoms due to the inter-atomic interactions. It will be shown that a collectiveness of the atoms emerges from the quantum correlations embedding in the wavefunctions with low-lying energy value, which is responsible for the unusual quantum behaviors. An artificial bipartition of the helium atoms is of little value for the pursuing of a reliable understanding of superfluidity.

In the past, several kinds of experimental observations contradicted the predictions of the two-fluid model in a direct way, which is not surprising given that the major part of the two-fluid model is of subjective nature.

Two experiments [15, 16] investigate the meniscus of superfluid  $^4\text{He}$  in a rotational bucket. According to the two-fluid model, only the normal component is rotating while the superfluid component remains static, therefore only normal component generate a meniscus and its height is proportional to the normal density. Consequently, the height of the meniscus shall increase with temperature and shall vanish at low temperature regime. But the experimental observation is just the opposite. The height of the meniscus does not change with the temperature and its value suggests that one hundred percents of the system is the normal component regardless of the temperature. The experiment [16] further reveals that this rotating  $^4\text{He}$  can demonstrate the thermo-mechanical effect, which is against the conclusion that one hundred percents of the system is the normal component.

Another type of experiments study the oscillation and the damping of superflow motion between two open vessels of superfluid  $^4\text{He}$  in the same isothermal enclosure [17, 18, 19, 20, 21, 22, 23]. If the liquid levels of two vessels are positioned differently, a superflow motion (typically in the form of mobile surface film) can be generated by gravitational potential difference; when the liquid levels are equalized, the inertial in the superflow subsequently cause the levels to oscillate about the equilibrium position. The dissipation of gravitational potential energy is present and damping of the oscillation of liquid levels is observed. It is interesting to study the temperature dependence of the damping rate of the oscillation. In the language of two-fluid model, the dissipation is caused by the normal component. Since the ratio of normal component approaches zero in the low temperature limit, the damping rate shall also vanish. But the experimental data shows the damping rate is finite and relatively large in the low temperature limit.

A similar behavior is also observed in the case of a superconductor. In an experiment [24, 25], the  $Q$  value of an microwave cavity made of superconductor niobium is measured. It is expected the  $Q$  value shall be exponentially increasing in the low temperature limit according to the two-fluid model. But the measurement shows the  $Q$

value approaches a constant value when  $T < 1.3K$ .

The dissipation behavior of superfluid  $^4\text{He}$  and that of a superconductor is regarded as an open question in the literatures. Later we shall show that it can be explained naturally. A superfluid system is not always dissipationless.

The two-fluid model implies that a  $^4\text{He}$  superflow carries zero thermal energy, which seems to be consistent with many experimental observations in the past. However, in those experiments the velocity of superflow ( $v_{sf}$ ) is not well controlled and the thermal energy density of a superflow is negligible at large  $v_{sf}$ . We have conducted an experiment where the value of  $v_{sf}$  is partially controlled. It is confirmed that the superflow can carry significant thermal energy, which gives rise to a counter-intuitive heating phenomenon (reported in Section V). This experimental observation is in a direct contrast to the pivotal hypothesis of the two-fluid model.

The above-mentioned problems of the two-fluid model indicate clearly that this model is not capable to provide a substantial description of superfluid  $^4\text{He}$ . It is necessary to develop a full quantum microscopic description of the superfluid  $^4\text{He}$  if one seeks a reliable, intrinsic and unified understanding of the system. Pursuing such a microscopic picture can be rewarding in other ways, such as leading to predictions of some new quantum phenomena.

### III. Grouping structure of microscopic levels of superfluid $^4\text{He}$

We consider a liquid  $^4\text{He}$  system which is translational invariant and periodic in the  $x$  direction and which is confined in the  $y, z$  directions by the inner wall of a container (see Fig. 1). the length of the system in the  $x$  direction is  $L$ , (i.e.,  $x + L$  is identical to  $x$ ). The Hamiltonian of the  $^4\text{He}$  system can be written as

$$\hat{H} = \sum_{i=1}^N -\frac{\hbar^2}{2M} \nabla_i^2 + \sum_{i<j}^N V(\mathbf{r}_i - \mathbf{r}_j), \quad (1)$$

where  $M$  is the mass of a  $^4\text{He}$  atom,  $\hbar$  is the reduced plank constant,  $N$  is the total number of the atoms and  $V$  is the interaction between two atoms.

The eigen-wavefunctions of the Hamiltonian operator satisfy the following equation

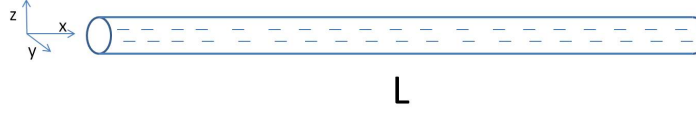
$$\hat{H}\psi_\alpha(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = E_\alpha\psi_\alpha(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N), \quad (2)$$

where  $\alpha$  labels the eigen-wavefunctions and  $E_\alpha$  is the eigen energy of the state.

Since  $\hat{H}$  commutes with the total momentum operator  $\hat{P}_x = -\hbar \sum_{j=1}^N \frac{\partial}{\partial x_j}$  (along the  $x$  direction), its eigen-wavefunctions can be the eigen-wavefunctions of the  $\hat{P}_x$  simultaneously

$$\hat{P}_x\psi_\alpha(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = P_\alpha\psi_\alpha(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N), \quad (3)$$

where  $P_\alpha$  is the eigen momentum.



**Figure 1.** The geometry of a liquid  $^4\text{He}$  system with a periodic length of  $L$  in  $x$  direction ( $x$  is identical to  $x + L$ ).

For a given eigen-wavefunction  $\psi_\beta(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  with an energy of  $E_\beta$  and momentum of  $P_\beta$ , one can construct some other eigen-wavefunctions by

$$\psi_\beta^{cm_k}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = e^{i \sum_{j=1}^N 2\pi k x_j / L} \psi_\beta(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N), \quad (4)$$

where  $k = \pm 1, \pm 2, \pm 3, \dots$ , is an interger.  $\psi_\beta^{cm_k}$  can be viewed as a Galilean transformation (center-of-mass motion) of the state  $\psi_\beta$ , with its energy and momentum given by

$$E_\beta^{cm_k} = E_\beta + (P_\beta + 2\pi k N \hbar / L)^2 / (2NM) - P_\beta^2 / (2NM) \quad (5)$$

and

$$P_\beta^{cm_k} = P_\beta + 2\pi k N \hbar / L. \quad (6)$$

Eq. 5 and Eq. 6 are due to the Galilean invariance of the system [11].

We shall study the properties of low-lying many-body levels of the system which are the relevant levels at low temperature, while the influence of high-lying levels is negligible due to the exponential Boltzmann factor.

A microscopic investigation of quantum transitions among the low-lying levels can lead to the revelation of a grouping structure among these levels. These quantum transitions are also responsible for the thermalization of the system. The microscopic atomic-molecular interactions between the system and its surroundings cause frequent momentum-energy exchange (between them) and drive the quantum transitions among the relevant levels of the system, which in turns leads to a thermal distribution of the levels.

The transition probability between two levels with an eigen-wavefunction of  $\psi_i(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  and an eigen-function of  $\psi_j(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$ , by the process of one-particle scattering, is determined formally by

$$P_{\psi_j \rightarrow \psi_i} = |\langle \psi_j(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) | \sum_{m,l} f(m-l) a_m^\dagger a_l | \psi_i(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) \rangle|^2, \quad (7)$$

where  $a_l$  is the annihilation operator corresponding to a single-particle orbit which carries an eigen momentum of  $2\pi l \hbar / L$  along  $x$ -direction (the wavefunction of the orbit has a  $x$ -dependence of  $e^{i2\pi l x / L}$ , the  $y, z$ -dependence of the orbit is ignored for simplicity),  $a_m^\dagger$  is the creation operator for a single-particle orbit with a wavefunction of  $e^{i2\pi m x / L}$ , and the function  $f(m-l)$  describes the strength of the scattering.

$P_{\psi_j \rightarrow \psi_i}$  can easily vanish depending on the two states. In some cases it can have an exact value of zero, while in the realistic case, it vanishes in the sense that it is



exponentially depressed when viewed as a function of  $N$ . Whether  $P_{\psi_j \rightarrow \psi_i}$  vanishes or not depends crucially on the ‘closeness’ of the two many-body wavefunctions. Take for example, if  $\psi_i$  assumes a form of Bose-Einstein condensation (BEC) which is accommodated by a single-particle orbit  $|\phi_1\rangle$  with a wavefunction of  $\phi(y, z)e^{i2\pi x/L}$ ,  $\psi_i(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = |\phi_1^N\rangle$  while  $\psi_j$  is the BEC of another single-particle orbit  $|\phi_2\rangle = \phi(y, z)e^{i4\pi x/L}$ ,  $\psi_j(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = |\phi_2^N\rangle$ , it is a simple exercise to demonstrate that  $P_{\psi_j \rightarrow \psi_i}$  vanishes. It is also easy to show that in the case of  $\psi_i(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = |\phi_1^{M_1} \phi_2^{N-M_1}\rangle$  (the notation means  $M_1$  particles occupying orbit  $\phi_1$  and  $N - M_1$  particles occupying orbit  $\phi_2$ ,  $M_1$  is an integer less than  $N$ ) and  $\psi_j(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = |\phi_1^{M_2} \phi_2^{N-M_2}\rangle$ ,  $P_{\psi_j \rightarrow \psi_i}$  always vanishes unless  $|M_1 - M_2| \leq 1$ . Here one can roughly consider the degree of difference between the state  $|\phi_1^{M_1} \phi_2^{N-M_1}\rangle$  and  $|\phi_1^{M_2} \phi_2^{N-M_2}\rangle$  as  $|M_1 - M_2|$ , which is the difference of number of particles occupying a major orbit. These simple observations suggest that if each of two quantum states involves a large number of particles to occupy a single-particle orbit, the transition probability between these states is negligible unless they have almost the same number of particles to occupy the same orbit.

We shall consider the ground state wavefunction of the system  $\psi_g(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$  which has the ground state energy  $E_g$  ( $\hat{H}|\psi_g\rangle = E_g|\psi_g\rangle$ ) and has a total momentum of zero along  $x$  direction ( $P_x|\psi_g\rangle = 0$ ). One can obtain another eigenstate which is a Galilean transformation of the ground state  $\psi_g^{cm1}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = e^{i\sum_{j=1}^N 2\pi x_j/L} \psi_g(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$ . The transition probability between two states is  $P_{\psi_g \rightarrow \psi_g^{cm1}} = |\langle \psi_g | \sum_{m,l} f(m-l) a_m^\dagger a_l | \psi_g^{cm1} \rangle|^2$ . It can be argued that  $P_{\psi_g \rightarrow \psi_g^{cm1}}$  vanishes naturally. First, the momentum of two states differ by  $2\pi N\hbar/L$ , which is large from a microscopic view point. In order to make such a transition, the single-particle scattering operator must correspond to orbits of very large single-particle momentums, but for the ground state, which can be written as a supposition of Fock states in momentum space, its component involving a single particle orbit of such large momentum is negligible, since the involvement of large momentum (single-particle) orbit costs high kinetic energy, which is unfavorable for the ground state. Therefore the scattering amplitude vanishes. Secondly, the ground state is naturally expected to be embedded with a strong correlation of the helium atoms. It is difficult to describe this correlation exactly, but a significant part of this correlation is manifested by a partial BEC in the wavefunction [8, 9, 10]. The condensate fraction of ground state is estimated to be around 10% in the literatures [26, 27, 28, 29, 30, 31]. For convenience, if a many-body eigenstate involves a partial BEC, the single particle orbit to accommodate this partial BEC is referred as the base orbit of this state. Obviously the base orbit of  $\psi_g$  has a zero eigen momentum along  $x$ -direction. The state  $\psi_g^{cm1}$  corresponds to an Galilean transformation of  $\psi_g$  and this transformation does not influence the internal correlations of the state. However, the base orbit of  $\psi_g^{cm1}$  has eigen momentum value of  $2\pi\hbar/L$ , and it is orthogonal to the base orbit of  $\psi_g$ . According to our discussion above, there is a large degree of difference between  $\psi_g^{cm1}$  and  $\psi_g$ , and consequently  $P_{\psi_g \rightarrow \psi_g^{cm1}}$  shall vanish.

Not only the ground state has this important type of correlation manifested in

a rough form of a partial BEC, but all low-lying eigen states possess the same type of correlation. This is largely due to the role of Bose exchange interaction [32]. If a quantum state involves a condensate fraction, the energy of Bose exchange interaction can be significantly lowered. On the other side, for an eigen state without a condensate fraction, its energy is raised substantially by the exchange interaction and it is excluded from the low energy regime.

Following the analysis above, a grouping structure of the low-lying levels emerges naturally. If two levels have the same base orbit, then they belong to the same group. Otherwise, they are in different groups.

The group which includes the the ground state  $\psi_g$  is denoted as  $grp_0$ . The group involves  $\psi_g^{cm1}$  is denoted as  $grp_1$ . It can be realized that  $grp_1$  can be viewed as a Galilean transformation of  $grp_0$ . Similarly,  $grp_k$  ( $k = 2, 3, \dots$ ) can be obtained by multiplying a Galilean factor of  $e^{i \sum_{j=1}^N 2\pi k x_j / L}$  to all eigen wavefunctions in  $grp_0$ . The group  $grp_{-|k|}$  with a negative integer value of  $k$  can be formed with a corresponding Galilean factor.

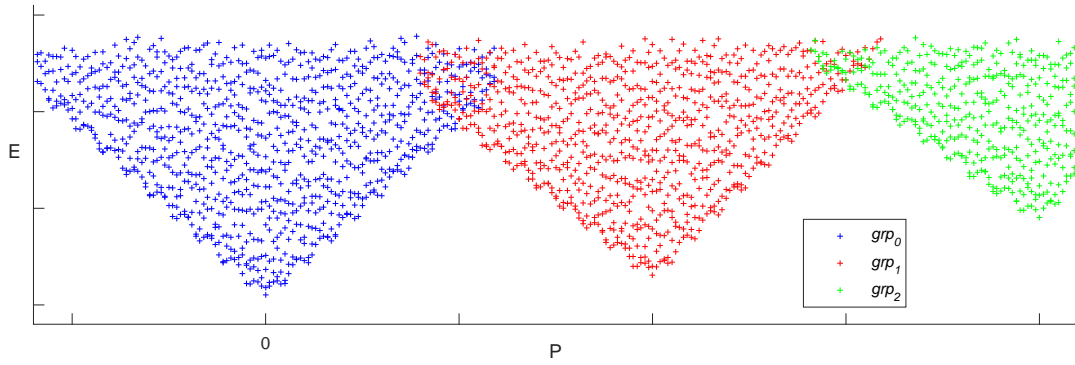
If two levels ( $\psi_a$  and  $\psi_b$ ) belong to the same group, one can alway find a chain of levels ( $\psi_n (n = 1, 2, \dots, N^c)$ ) in this group so that the transition probalities  $P_{\psi^a \rightarrow \psi_1}, P_{\psi_1 \rightarrow \psi_2}, P_{\psi_2 \rightarrow \psi_3}, \dots, P_{\psi_{N^c-1} \rightarrow \psi_{N^c}}, P_{\psi_{N^c} \rightarrow \psi_b}$  do not vanish. In other words, there is a chain of single-particle scattering process which can link  $\psi_a$  to  $\psi_b$ . If two levels belong to the different groups, one cannot find a long chain of scattering process which links these two levels, unless some high-lying levels are involved.

At low temperature regime, the system can establish a group-specific thermal equilibrium with its surroundings. For example if some levels in a group is initially occupied, then the frquenct quantum exchanges between the system and its surroundings shall cause the level occupations to spread out, and eventually a thermal distribution of all levels in the group is reached. The initially-unoccupied groups remain unoccupied since the inter-group transition is prohibited by high energy barriers.

The grouping structure of the low-lying levels is the quantum root of many fascinating behaviors of superfluid  $^4\text{He}$ , and one cannot overemphasize the importance of this grouping structure.

In Fig.2, some levels in  $grp_0$ ,  $grp_1$  and  $grp_2$  are schematically plotted in the momentum energy plane. The lower boundaries (the many-body dispersion lines ) of  $grp_0$  are roughly linear in both positive and negative momentum directions and the slopes of two linear dipserions have the same magnitude [32, 33, 7, 11]. In  $grp_1$ , the slope magnitude of positive linear dispersion is larger than slope magnitude of negative linear branch, this is due to that Galilean transformation of a positive linear dispersion line (in the positive  $P$  direction) will become more titled while it become less titled in the case of negative linear dispersion. The slope magnitude of the positive linear dispersion line increases furthers in the case of  $grp_2$  in the figure. Note that  $grp_0$  and  $grp_1$  are overlapped in some regions in this plot and they will become more overlapped in the regions of high energy. The overlapping of two groups might cause a false impression that quantum jumps between two groups are possible. As analyzed above, every eigen wavefunction in  $grp_0$  has a large degree of difference from any eigen wavefunction in





**Figure 2.** A schematic plot of some low-lying levels of  $grp_0$ ,  $grp_1$  and  $grp_2$  (labeled in color) in the momentum energy plane. Each level is marked by a plus sign. The groups overlap in some regions in the plot, and they overlap more in regions of high energy (not shown in the figure).

$grp_1$  due to that they have different base orbits, therefore transition probability between them vanishes.

The phenomenon of superfluidity can be naturally explained in the light of the grouping structure of the levels. At low temperature, the system establishes a group-specific thermal equilibrium with its surroundings. The initial occupied group(s) of levels remains occupied while the initially un-occupied groups keep being empty. If only the group of  $grp_0$  is initially occupied, the average momentum of the system is zero at thermal equilibrium, corresponding to a (macroscopically) static state of the system. Assume that  $grp_2$  is initially occupied only, the average momentum of the system at thermal equilibrium is determined by

$$\bar{P}_{grp_2}^T = \frac{\sum_{\phi_\gamma \in grp_2} \langle \phi_\gamma | \hat{P}_x | \phi_\gamma \rangle e^{\frac{-E_\gamma}{kT}}}{\sum_{\phi_\gamma \in grp_2} \langle \phi_\gamma | \phi_\gamma \rangle e^{\frac{-E_\gamma}{kT}}} \quad (8)$$

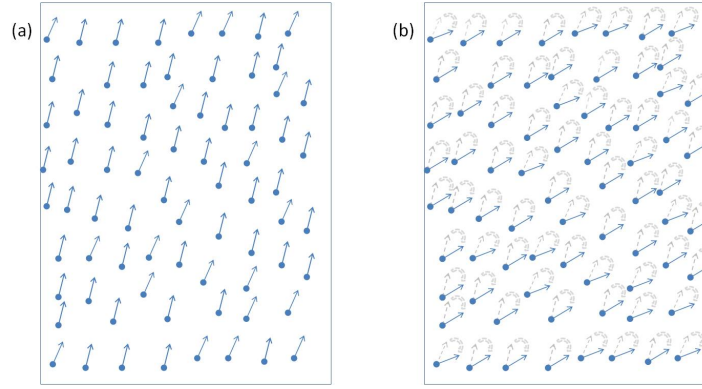
$\bar{P}_{grp_2}^T$  has a positive finite value, which indicates that a finite persistent current is present in the system.  $\bar{P}_{grp_2}^T$  has a temperature dependence due to the Boltzmann factor in Eq. 8. If the temperature changes gradually and returns to its initial value,  $\bar{P}_{grp_2}^T$  varies and returns to its initial value accordingly. Such a temperature dependence is largely confirmed by an experiment [34] in the past.

In reality, the translational invariance of superfluid  $^4\text{He}$  is not perfect. Consider for example the inner wall of the container has a degree of roughness which shall break the translational symmetry. But such symmetry-broken interaction (potential) can be considered as a small perturbation term compared with terms presented in Eq.1 (The atomic-molecular interactions between the system and the wall of the container are restricted to a few layer of helium atoms close to wall, and the interaction strength drops

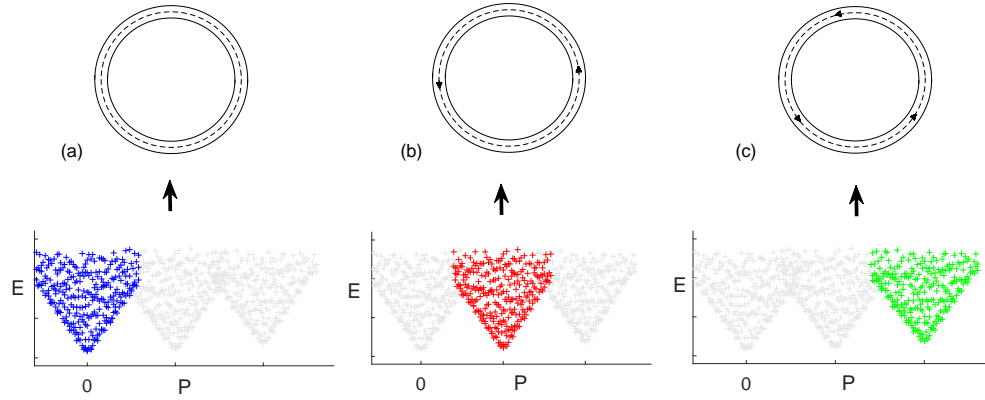
quickly if a helium atom is away from the wall surface). With this perturbation term, the microscopic many-body wavefunctions are not eigenstates of  $\widehat{P}_x$ , one shall replace the eigenvalue by the expectation value of this operator for theoretical considerations. This perturbation term might mix the levels of the same group by an infinitesimal amount (in the high order treatments), but it is not sufficient to mix two levels which belong to two different groups. Inter-group mixture is only possible if the perturbation term is capable of exert a homogeneous influence on all atoms in the system. The grouping structure of microscopic levels is quite robust regardless of an exact translational symmetry.

The grouping structure of superfluid  $^4\text{He}$ 's low-lying levels is of full quantum nature as it is related to a particular property of each low-lying level (the corresponding wavefunction has a condensate fraction) and to the property of transition probabilities among these levels. Although it was unnoticed in the past, one can realize that grouping of microscopic levels is not unique to a superfluid system. Consider a ferromagnet for instance. Below the Curie point, the microscopic magnetic moments in the ferromagnet point roughly to the same direction (see Fig.3(a)). The system has many macroscopic states since there are many different directions of total magnetic moments. Each macroscopic state can be considered as a thermal equilibrium state which is group-specific (a specific group of microscopic levels is involved). Different macroscopic states correspond to different groups of levels. The metastability of each macroscopic state is microscopically due to that: i) the thermal transition between any two different groups of microscopic levels involves a high energy barrier which separates the groups, therefore this type of transition is prohibited at low temperature. ii) A smooth transition between two different groups of levels is possible if all spins rotate by the same amount of angle simultaneously (see Fig.3(b)). This collective motion of all spins does not encounter a high energy barrier. However, this collective motion cannot be initiated spontaneously and it must involve some external interference such as the application of a strong magnetic field. One shall note that, in the case of superfluid  $^4\text{He}$ , a smoother transition between different groups of levels can be established when there is a global force (for example the gravity or a hydrodynamic pressure force) which is applied to all helium atoms and accelerate them in a collective form.

In the light of the analogy between the superfluid  $^4\text{He}$  and a ferromagnet, one can view the grouping structures of the microscopic levels from a broader perspective. There are two fundamental facts regarding this kind of systems : i) each system exhibits a number of macroscopic metastable states at low temperature regime; and ii) it has a large number of microscopic low-lying levels. Establishing an inherent connection between two facts shall naturally lead to the realization that the low-lying levels of the system fall into a grouping structure, and that different groups of levels are physically separated by some high energy barriers. In this way, A macroscopic metastable state corresponds to a group of microscopic levels naturally (such a macroscopic- microscopic correspondence resolves the mystery of superfluidity in a transparent way, see Fig.4). On the other hand, if temperature is high enough, the energy barriers are overcome and all groups of low-lying levels will be thermally occupied together with the relevant



**Figure 3.** (a) In a ferromagnet, the microscopic magnetic moments (spins) point to the same direction roughly. (b) A smooth transition between two metastable states is possible if all microscopic moments rotate in a collective manner, which requires an external manipulation in general.



**Figure 4.** A schematic plot of macroscopic-microscopic correspondence behind the metastabilities of  $^4\text{He}$  superflows. a) a static state. b) a superflow state with a small velocity. c) a superflow state with a velocity slightly larger than the velocity in (b). The occupied group is highlighted in color.

high-lying levels, thus the number of the macroscopic thermal states is reduced to be one.

These discussions can extend to other condensed matter systems which possess low-temperature phases described by an order parameter. The microscopic quantum pictures of these systems are generic with respects to the following aspects: i) there exists a grouping structure of the low lying microscopic levels in each system. When one specific group is thermally occupied while the other groups are empty, the system demonstrates a corresponding metastable state. The value of the order parameter is determined by the occupied group statistically. ii) A smooth inter-group transition at

low temperature might be possible if there is a proper type of collective motion involving all particles in the system. Such a collective motion is far beyond being unconditional. An inter-group transition via a chain of spontaneous thermal excitation of the system is not possible because it involves high-lying levels which are effectively irrelevant at low temperature. iii) When the temperature is raised above the transition point, the grouping of low-lying levels is not applicable any more. Not only some thermal excitation processes involving the now-reachable high-lying levels can make inter-group transitions possible, but also these high-lying levels (which do not belong to any the low-lying groups) are more abundant and statistically more significant (than the low-lying levels). Hence, the system restores to a single thermal state with an order parameter value of zero at high temperature.

We shall explain why superfluid  $^4\text{He}$  is not always dissipationless in some cases. Some relevant experiments were mentioned in Section II. The dissipationless behavior is available only if the superflow is in a state of a group-specific thermal equilibrium which corresponds to a steady flow motion. When a superflow is flowing back and forth between two vessels, the microscopic inter-group transitions are taken place frequently due to a global force which drives all helium atoms in the superflow. Many groups of levels are occupied and unoccupied in time and the superflow is not in a state of thermal equilibrium. Dissipation is naturally present in these microscopic transition processes. When superfluid  $^4\text{He}$  is subject to a time-varying motion, its dissipation behavior is similar to that of a normal system in terms of the temperature dependence of the dissipation rate at low temperature limit.

## VI. Velocity dependence of the thermal energy of a superflow

The previous section illustrated the the grouping structure of microscopic levels of superfluid  $^4\text{He}$ . It is the clear that the macroscopic physical properties of the system are determined by the occupied group(s) in a statistical manner and have a dependence on the group(s) naturally. We shall consider two important properties: the superflow velocity and the thermal energy.

One can define a velocity parameter  $v_k$  to further characterize the group  $grp_k$  as

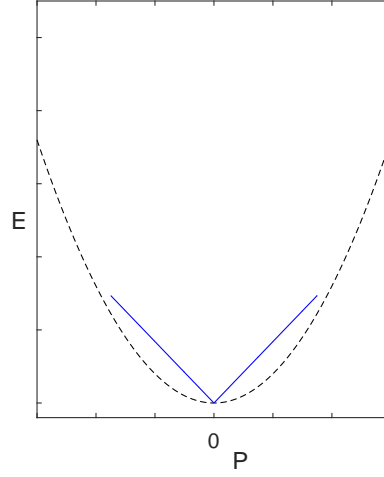
$$v_k = 2\pi\hbar k / ML. \quad (9)$$

It corresponds to the center-of-mass velocity of the microscopic state with a wavefunction of  $e^{\sum_{j=1}^N i2\pi k x_j / L} \psi_g$ , which belongs to the group  $grp_k$ .

The thermally averaged velocity of the system in the group of  $grp_k$  is determined by

$$\bar{v}_{grp_k}^T = \frac{\sum_{\phi_\gamma \in grp_k} \langle \phi_\gamma | \widehat{\frac{P_x}{NM}} | \phi_\gamma \rangle e^{-\frac{E_\gamma}{kT}}}{\sum_{\phi_\gamma \in grp_k} \langle \phi_\gamma | \phi_\gamma \rangle e^{-\frac{E_\gamma}{kT}}}. \quad (10)$$

It is obvious that  $\bar{v}_{grp_k}^T$  corresponds to the actual superflow velocity. Its value has a temperature dependence and can differ largely from the value of  $v_k$ . Realistic



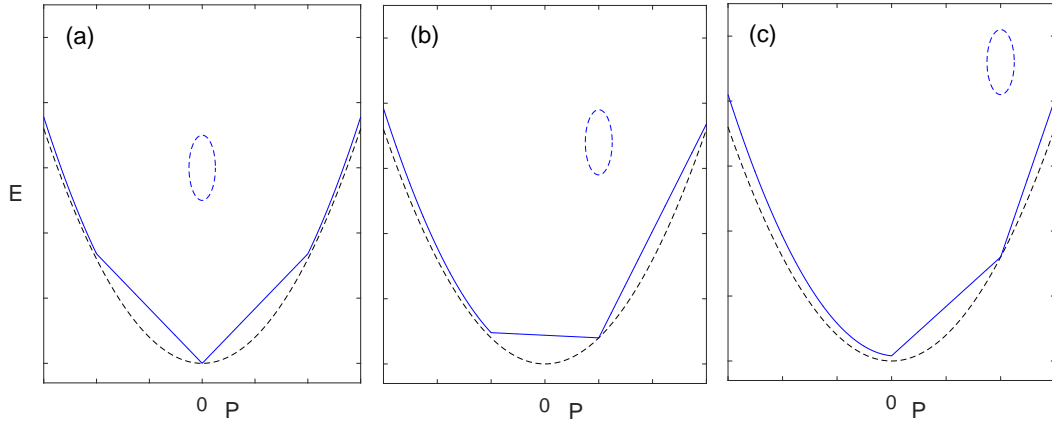
**Figure 5.** Blue lines are the linear dispersions of  $grp_0$ . The dashed parabola corresponds to  $E(P) = E_g + P^2/2NM$ . Since all levels of the system should be located above the parabola, the blue lines cannot extend further to cross the curve. The dispersion behavior has to be adjusted at large  $|P|$  regions so that extended dispersion lines can remain above the parabola (see Fig.6 (a)).

considerations of superfluid  $^4\text{He}$  suggest that  $v_k$  can be as large as of several tens of  $m/s$  or even above, while the corresponding  $\bar{v}_{grp_k}^T$  (at a temperature well below the transition point) is roughly smaller by two orders of magnitude.  $\bar{v}_{grp_k}^T$  approaches a zero value near the transition point. At a given temperature (above  $1K$ ),  $\bar{v}_{grp_k}^T$  increases with the group number  $k$  and it be used to label the groups. In this way, one can view other group-dependent properties of the system as being flow-velocity dependent. We shall investigate how the flow-velocity dependence of the superflow's thermal energy.

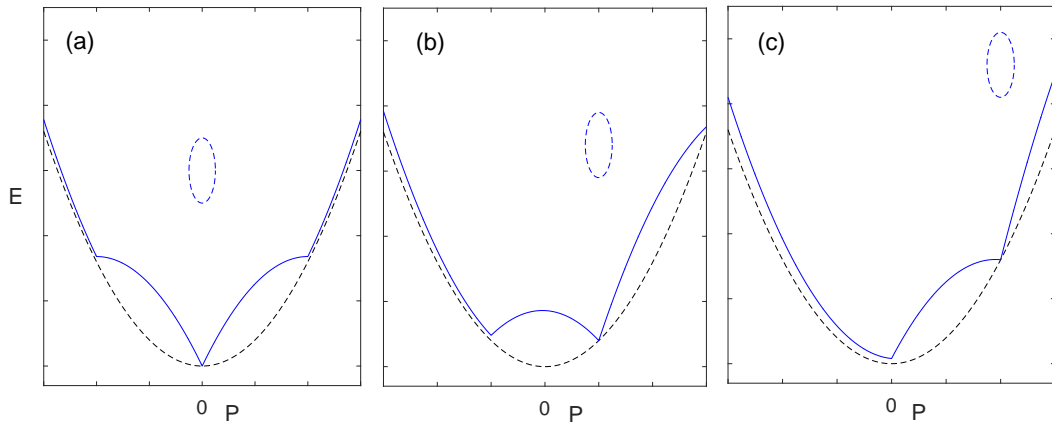
The thermal energy of the system in the group of  $grp_k$  is formally calculated as

$$\bar{E}_{grp_k}^T = \frac{\sum_{\phi_\gamma \in grp_k} \langle \phi_\gamma | E_\gamma | \phi_\gamma \rangle e^{-\frac{E_\gamma}{kT}}}{\sum_{\phi_\gamma \in grp_k} \langle \phi_\gamma | \phi_\gamma \rangle e^{-\frac{E_\gamma}{kT}}}. \quad (11)$$

In order to have a qualitative analysis of  $\bar{E}_{grp_k}^T$ , one shall have a full picture of how the levels in  $grp_k$  are distributed in the momentum-energy plane. It is sufficient to investigate the level distributions of  $grp_0$  since  $grp_k$  can be treated as a Galilean transformation of  $grp_0$ . In Fig.2, only a small part of levels  $grp_0$  is schematically plotted with the lower boundaries are composed of two roughly linear dispersions. This linear behaviors of the boundaries shall be modified at large momentum regimes for a fundamental reason. In Fig.5, the linear boundaries of  $grp_0$  are plotted together with a parabolic curve which is defined as  $E = E_g + P^2/2NM$ . It can be realized that any microscopic levels of the system cannot be below this parabolic curve. One can use a proof-by-contradiction method to illustrate this point. If there exists an eigenstate (described by the wavefunction  $\psi_\zeta$ ) which has a momentum of  $P_\zeta$  and has an energy  $E_\zeta < E_g + P_\zeta^2/2NM$ , one can find an integer  $l$  such that  $l \leq -\frac{P_\zeta L}{2\pi\hbar} < l+1$ , then the

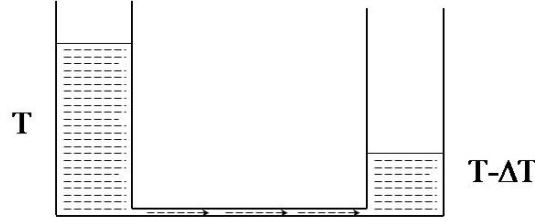


**Figure 6.** (a) The slope magnitude of the linear dispersion line of  $grp_0$  remains constant. The parabola (dashed) corresponds to  $E(P) = E_g + P^2/2NM$ . (b) and (c) are the two Galilean transformations of  $grp_0$ , corresponding to a superflow state with a middle flow velocity and with a large flow velocity, respectively. The ellipse in (a) denotes roughly the region of the dense levels which make the major contribution of the thermal energy of the system (at a given temperature). The ellipse region is raised in (b) and (c) and the dense levels in the region are less involved in the contribution of the thermal energy, resulting into the decreasing of the thermal energy.



**Figure 7.** In this figure, the dispersion of  $grp_0$  is different from that in Fig.6(a). The dispersion is linear at small momentum region but the slope magnitude decreases with  $|P|$ . (b) and (c) are the two Galilean transformations of  $grp_0$ .





**Figure 8.** Mechano-caloric Effect of superfluid  $^4\text{He}$ . The superflow in the narrow channel has a large flow velocity and a small thermal energy density. Upon exiting the channel, the superflow adapts to a state of vanishing flow velocity and it has a large thermal energy density if the temperature keeps the same. The conservation law of energy requires a drop of the temperature to compensate the change of energy attributed to the flow velocity.

state described by the wavefunction  $\psi_\zeta^{cm_i} = e^{\sum_{j=1}^N i2\pi l x_j / L} \psi_\zeta$  will have a momentum very close to zero and it has an energy roughly  $E_\zeta^{cm_i} = E_\zeta - P_\zeta^2 / 2NM$  which is smaller than  $E_g$ . This is a contradiction since the lowest eigen energy should be  $E_g$ .

Fig.6(a) shows one possible scenario of the dispersion behavior of  $grp_0$  where the slope magnitude of the linear dispersion remains a constant change until it is close to the parabola. Fig.7(a) shows another possible scenario of dispersion behavior, where the slope magnitude of linear dispersion decreases with  $|P|$  until approaching the parabola.

After illustrating the qualitative behavior of the low boundaries of  $grp_0$  at the momentum-energy plane, one shall pay attention to another important factor concerning the analysis of thermal energy, which is the level density of  $grp_0$  at different regions in the momentum energy plane. It can be naturally figured out that levels are relatively sparse in the region which is close to low boundary of the group, while levels are much more dense in the middle region around  $P \approx 0$  and with proper high energy. Taking into account of Boltzmann factor  $e^{-E/kT}$ , one shall realize the levels in the region enclosed roughly by the ellipse in Fig.6(a) (and in Fig.6(a)), contribute mostly to the thermal energy of the system while contributing from the (sparse) levels in the rest region is relatively small. If the ellipse region of the dense levels is excluded, the thermal energy will be decreased substantially.

When  $grp_0$  is mapped into  $grp_k$  (see Fig.6 and 7), one then realizes that region of the high level-density is moved to right and is also lifted roughly by an energy amount of  $\Delta E = 1/2NMv_k^2$ , according to Eq. 5. In the mean time, the sparse regions close to the left boundary of  $grp_0$  are shifted to have much lower energies. The lift of the high level-density region leads to its diminishing contribution to the thermal energy due to the Boltzmann factor in Eq.11, and the value of  $E_{grp_k}^T$  is reduced for the decreasing of its major part.  $E_{grp_k}^T$  is a decreasing function of  $|v_k|$  or  $|k|$  due to the lift of the high level-density region.  $\Delta E$  is  $1.3 \times 10^{-24} J$  per an atom at  $v_k = 20 m/s$ , and it is  $12 \times 10^{-24} J$  per an atom at  $v_k = 60 m/s$ . The average thermal energy of (static) superfluid  $^4\text{He}$

is around  $2.6 \times 10^{-24} J$  per an atom at  $T = 1.6 K$  and  $5.6 \times 10^{-24} J$  per an atom at  $T = 1.8 K$ . The comparisons of these numbers can suggest that the high level-density region is effectively negligible in its contribution to the thermal density when  $v_k$  is large enough. The thermal energy density of a superflow can decrease by orders of magnitude with increasing flow velocity, which is an outstanding quantum property of superfluid  $^4\text{He}$ .

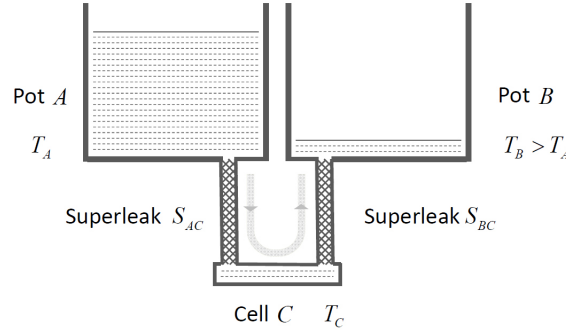
The mechano-caloric effect of superfluid  $^4\text{He}$  can be explained naturally in the light of this velocity-dependence of the thermal energy (see Fig.8). If the flow velocity of a superflow is subject to a significant variation, the thermal energy is largely affected. The conservation law of energy requires the temperature of the superflow varies accordingly to compensate the change of energy attributed to the flow velocity. This intrinsic coupling between the variation of flow velocity and the variation of temperature is a reflection of the fact the thermal state of the system is group-specific and depends on the occupied group below the transition point.

## V. Experimental observation of an self-heating effect of $^4\text{He}$ superflows

In this section, we present an experimental observation of an self-heating phenomenon of  $^4\text{He}$  superflows, which confirms that  $^4\text{He}$  superflows can carry significant thermal energies. This counter-intuitive heating effect bears a phenomenological resemblance to the Peltier effect of electric current across two different conductors.

The main setup of the superflow system is schematically plotted in Fig.9. Three vessels (referred to as pot  $A$ , pot  $B$  and cell  $C$ ) are connected in series by two superleaks (referred to as  $S_{AC}$  and  $S_{BC}$ ). Cell  $C$  is thermally isolated from its surroundings, except its thermal links to the pots via the superleaks. In the experiment, pot  $A$  is filled with superfluid  $^4\text{He}$  initially, and superflows through  $S_{AC}$ , cell  $C$  and  $S_{BC}$  can be established eventually by setting a positive temperature difference between pot  $B$  and  $A$  (*i.e.* fountain effect), resulting in a superfluid transport from pot  $A$  to  $B$ . In general, one would expect that the temperature of cell  $C$  ( $T_C$ ) shall eventually lie between the temperature of pot  $B$  ( $T_B$ ) and that of pot  $A$  ( $T_A$ ). However, it is observed that cell  $C$  can be heated strikingly by the superflows in some cases, and consequently  $T_C$  reaches a steady value which exceeds  $T_B$  by more than one hundred millikelvins.

The experiment is carried out on a two-stage Gifford-McMahon refrigerator with a cooling power of  $1 W$  at  $4.2 K$  and a base temperature of  $2.4 K$ . In order to reach the superfluid temperature regime, a liquid  $^4\text{He}$  cryostat is constructed basically according to the scheme given in Ref. [35]. A stainless steel capillary, with an inner diameter (i.d.) of  $0.18 mm$ , an outer diameter (o.d.) of  $0.4 mm$  and a length of  $1 m$ , is used as the Joule-Thomson impedance in the cryostat. The copper pot for collecting liquid  $^4\text{He}$ , with an i.d. of  $4.0 cm$  and a volume of  $78 cm^3$ , is also served as pot  $A$  for the experiment. Another copper pot, identical to pot  $A$ , is used as pot  $B$ . Cell  $C$  is made of a small copper block, and the main part of its inner cavity is cylindrical, with a diameter of  $3 mm$  and a length of  $40 mm$ . Each superleak is made of a stainless steel tube packed with



**Figure 9.** A schematic plot of superflow system.



**Figure 10.** A picture of superflow system.  $S_{AC}$  and  $S_{BC}$  refer to the superleaks.

jeweler's rouge powder (with an average particle size of  $70\text{ nm}$  determined by TEM). The tube for  $S_{AC}$  has an i.d. of  $0.8\text{ mm}$ , an o.d. of  $2.0\text{ mm}$  and a length of  $65\text{ mm}$ , while the tube for  $S_{BC}$  has an i.d. of  $1.0\text{ mm}$ , an o.d. of  $2.0\text{ mm}$  and a length of  $65\text{ mm}$ . Two superleaks are soft soldered to cell  $C$ , and they are positioned in a way so that the lower end of each superleak is in proximity to one end of cell  $C$ 's cylindrical cavity (see Fig.10). The upper end of  $S_{AC}$  joins pot  $A$  while the upper end of  $S_{BC}$  joins pot  $B$ .

A combination of copper braids and brass strips is used as a thermal link between pot  $A$  and a cooling plate directly mounted to the second stage of the refrigerator, with a thermal conductance of around  $2\text{ mW/K}$  at  $2\text{ K}$ . Pot  $B$ 's major thermal link with its surroundings is a copper braid joining two pots at ends. Resistance wires wrapped around pots are used as heaters. Pot  $B$  as well as pot  $A$  is equipped with a pumping line. Valves are used in the lines so that the pumping rate can be manipulated to provide further means for the temperature controls of the pots (The values of  $T_A$  and  $T_B$  are well controlled during the superfluid transport stage which last for hours in the experiment). Calibrated carbon ceramic resistances [36] are used as temperature sensors to measure

$T_A$ (K)	$T_B$ (K)	$T_C$ (K)
1.500(4)	1.700(4)	1.847 (1)
1.600(4)	1.800(4)	1.927 (1)
1.600(4)	1.900(4)	2.014 (1)

**Table 1.** Steady values of  $T_C$  at given  $T_A$  and  $T_B$ . The number in the parenthesis denotes the fluctuation amplitude of the measured temperature in a period of several hours.  $T_C$  is quite stable and its deviation is smaller than  $1\text{mK}$ .

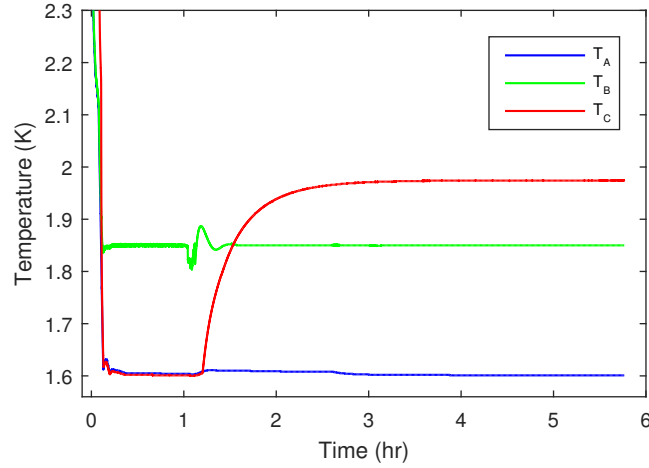
$T_A$ ,  $T_B$  and  $T_C$ , with an accuracy of  $5\text{ mK}$ . The dissipation power of temperature sensor on cell  $C$  is kept much below  $10^{-7}\text{W}$ , so that its heating effect is very limited.

Two radiation shields made of copper coated with thin nickel layer are amounted to two stages of the refrigerator, with one at a temperature around  $45\text{K}$  and the other at a temperature below  $2.8\text{K}$ . Great care is made to prevent cell  $C$  from exposing to the thermal radiations from sources with a temperature above  $3\text{K}$ .

For the initial accumulation of liquid  $^4\text{He}$  (with a purity of 99.999%) in pot  $A$ ,  $T_A$  is intentionally raised above  $\lambda$  point to prevent superflow through  $S_{AC}$ . Cell  $C$  and pot  $B$  remain empty at the initial stage. After pot  $A$  is filled with a large amount of liquid  $^4\text{He}$ , its temperature is lowered by adjusting the pumping rate. Once  $T_A$  is below  $\lambda$  point, a superflow through superleak  $S_{AC}$  starts to fill cell  $C$ . The value of  $T_B$  is set to be well above the set value of  $T_A$  and superfluid transport from pot  $B$  to pot  $A$  via cell  $C$  can be established eventually. This superfluid transport continues for several hours without any disruption and it is found that  $T_C$  is quite stable after reaching its steady value. Some steady values of  $T_C$  at given  $T_A$  and  $T_B$  are listed in Tab. 1.

A heating process of cell  $C$  is plotted in Fig.11. At the initial moment represented in the figure, pot  $A$  is filled with liquid  $^4\text{He}$  and has a temperature above  $\lambda$  point while both pot  $B$  and cell  $C$  are empty. Pumping pot  $A$  leads to dropping of  $T_A$ .  $T_B$  drops in pace with  $T_A$  due to relatively large thermal link between two pots. At some point, electric current of the resistance wire around pot  $B$  is activated to stabilize  $T_B$  at the set value of  $1.85\text{K}$ .  $T_C$  decreases with  $T_A$ . When both of them are getting below  $\lambda$  point, superfluid transport between pot  $A$  and cell  $C$  is initiated and consequently plays a major role in determining the value of  $T_C$  relative to  $T_A$ .  $T_C$  remains a few millikelvins below  $T_A$  for superfluid filling duration of cell  $C$ . If  $T_C$  is further below, fountain pressure of the superfluid, caused by the difference between  $T_A$  and  $T_C$ , overcomes the gravitational pull and directs superfluid back from cell  $C$  to pot  $A$ , leading to increase of  $T_C$ . On the other hand, if  $T_C$  is getting closer to or further above  $T_A$ , the overall force (fountain pressure plus the gravitational pull) conducts superflow from pot  $A$  to cell  $C$ , resulting in decrease of  $T_C$ . This negative feedback mechanism of temperature locks roughly the value of  $T_C$  relative to  $T_A$ .

When cell  $C$  is approximately fully filled and superfluid transport between cell  $C$  and pot  $B$  is initiated,  $T_B$  goes through some large variation, it goes down and up for several times in a period of around half an hour. This is mainly due to the fact that



**Figure 11.** A heating process of cell  $C$ .

heat capacitance of the empty pot  $B$  (made of copper) is relatively small. Even a small amount of cold superfluid  $^4\text{He}$  entering pot  $B$  can cause a large variation of  $T_B$  and the temperature stabilization system is not capable to respond in a timely manner. After a certain amount of superfluid  $^4\text{He}$  is accumulated in pot  $B$ , further injection of cold superfluid is no longer capable to change  $T_B$  dramatically and the stabilization of  $T_B$  is well restored.

Once the transport of superfluid  $^4\text{He}$  through two superleaks is fully established,  $T_C$  rises steadily and reaches a value 120  $mK$  above  $T_B$ , which demonstrates an counter-intuitive heating phenomenon. The data presented in Fig.11 can provide useful estimations of the heating power received by cell  $C$  at two different stages. The first stage corresponds to the filling of superfluid into cell  $C$  which takes approximately an hour. At this stage, cell  $C$  receives heat transfer from pot  $B$  by thermal conduction while it is cooled by the influx of superfluid (note that this heat transfer drives superfluid to flow into cell  $C$ ). The heat received by cell  $C$  at this stage corresponds to the thermal energy of liquid  $^4\text{He}$  with a volume of 0.1  $c.c$  at temperature of 1.6  $K$ , and the heating power (denoted by  $P_{stage1}$ ) is around 1.5  $\mu W$ . Right after the first stage, the temperature of cell  $C$  (superfluid filled) rise from 1.6  $K$  to 1.85  $K$  in a duration of 20 minutes and we consider this duration as the second stage. The heat received at this stage corresponds roughly to the change of thermal energy of liquid  $^4\text{He}$  in cell  $C$  when temperature rised to 1.85 $K$  from 1.6 $K$ . The heating power at the second stage (denoted by  $P_{stage2}$ ) is 7.3  $\mu W$ . Consider that even if cell  $C$  could receive some abnormal background heating in the experiment, this heating should be present also in the first stage and therefore has a power less than  $P_{stage1}$ . As the background heating is not sufficient to explain the heating effect at the second stage given that  $P_{stage2} > 4P_{stage1}$ , it is concluded that the major portion of  $P_{stage2}$  has to be sourced from the superflows and it is responsible for the heating of cell  $C$  for the rest of time plotted in Fig.11. Since the kinetic energy of the superflows is negligible [37], this heating by superflow must take origin from

the thermal energy of superflows. This heating phenomenon is some way analogous to the Peltier effect of an electric current flowing across two conductors: the superflow entering cell  $C$  (inlet superflow) carries a thermal energy (density) larger than that of the superflow leaving from cell  $C$  (outlet superflow), leading to the heating of cell  $C$ . The velocity dependence of a superflow's thermal energy is responsible for generating such a difference between the inflow and the outflow.

The superflow velocities in the superleaks behave in a rather complicate manner. Note that a superflow is frictionless and its velocity cannot be stabilized by friction, which is in contrast to the case of an ordinary flow. The superflow in superleak  $S_{AC}$  keeps accelerating or decelerating, subject to pressure difference between superfluid in pot  $A$  and that in cell  $C$  (the fountain pressure, caused by the temperature difference across  $S_{AC}$ , consists of a significant part of the overall pressure difference). The pressure in cell  $C$  often rises abruptly as it changes from a nearly-fully-filled state to a fully-filled state frequently. The sudden rise of pressure regulates the inlet superflow by a great deal and prevents it from obtaining a rather large velocity. On the other hand, the outlet superflow can be accelerated with the pressure rise in cell  $C$  and can reach large velocity regime. Moreover, with  $T_C > T_B$ , the superflow in  $S_{BC}$  can reverse flow direction when cell  $C$  deviates from a fully-filled state, thus the actual flow time of outlet superflow is shorter than the flow time of inlet superflow. From these analyses it is clear that there is an asymmetry between the velocity distribution of the inlet superflow and that of the outlet superflow, which in turn leads to a difference between their thermal energies.

We also conducted a comparative experiment in which the superleak  $S_{BC}$  is replaced by a solid rode of stainless steel of the same size (a radius of  $2.0\text{mm}$  and a length of  $65\text{mm}$ ). In this experiment, superfluid transport can only take place between pot  $A$  and cell  $C$  while it is blocked between cell  $C$  and pot  $B$ . It is expected and confirmed that the steady value of  $T_C$  is well between  $T_A$  and  $T_B$ . The comparison between two experiments gives another solid evidence for the conclusion that the unusual heating phenomenon is not due to some abnormal background heating.

Many experiments in the past seem to suggest  $^4\text{He}$  superflows carry negligible thermal energies. This is attributed to the fact that the superflow velocity is not controlled and its value often approaches the critical value in the experiments. In the constrast, the experiment reported here involves a partial control of the superflow velocities which leads to an intriguing heating phenomenon.

## VI. Conclusions

In this paper, we established a direct correspondence between the various macroscopic phenomena of superfluid  $^4\text{He}$  and the microscopic properties of its quantum many-body levels. We showed that the relevant low-lying levels form a natural grouping structure and the system can establish a group-specific thermal equilibrium with its surroundings as inter-group transitions are prohibited at low temperature. This group-specific thermal equilibrium gives rise to a non-zero average value of momentum along a flow direction



and it is manifested as a superflow state macroscopically.

We also showed that the thermal energy (density) of a superflow depends on the occupied group, which is further transformed into a flow-velocity dependence: the larger the velocity is, the smaller the thermal energy. This rather unusual dependence is responsible for the intriguing coupling between the system's flow motion and its thermal motion.

According to the principle of quantum mechanics, all observed properties of superfluid  $^4\text{He}$  are just reflections of properties of its underlying many-body quantum states. Once some pivotal properties of its quantum states are figured out, it is not surprising that many unusual behaviors of this quantum system can be understood in a straightforward manner.

Partially motivated by the theoretical study, we conducted an experiment of  $^4\text{He}$  superflows and observed a counter-intuitive heating phenomenon, which confirms that a superflow can carry significant thermal energy, depending on the flow velocity.

Many experiments can be further carried out in the light of the theoretical picture presented in this paper. For example, Experiments with a full control of superflow velocity can be designed and the velocity dependence of the thermal energy can be investigated in a quantitative way.

## References

- [1] P. Kapitza, *Nature* **141**, 74 (1938).
- [2] J. F. Allen & A. D. Misener, *Nature* **141**, 75 (1938).
- [3] J. F. Allen & H. Jones, *Nature* **141**, 243 (1938).
- [4] J. G. Daunt & K. Mendelssohn, *Nature* **143**, 719 (1939).
- [5] L. Landau, *J. Phys. USSR* **5**, 71 (1941).
- [6] L. Landau, *J. Phys. USSR* **11**, 91 (1947).
- [7] N. Bogoliubov, *J. Phys. USSR* **11**, 23 (1947).
- [8] F. London, *Nature* **141**, 643 (1938).
- [9] F. London, *Phys. Rev.* **54**, 947 (1938).
- [10] L. Tisza, *Nature* **141**, 913 (1938).
- [11] F. Bloch, *Phys. Rev. A* **7**, 2187 (1973).
- [12] A. G. Leggett, *Rev. Mod. Phys.* **73**, 307 (2001).
- [13] R. Feynman, R. B. Leighton & M. L. Sands, *The Feynman Lectures on Physics*, vol. III, chapter 4, Addison-Wesley, Reading, MA (1965).
- [14] D. R. Tilley & J. Tilley, *Superfluidity and Superconductivity*, A. Hilger, 1986.
- [15] D.V. Osborne, *Proc. Phys. Soc. A* **63**, 909 (1950).
- [16] E. L. Andronikashvili & I. P. Kaverlin, *J. Exp. Theor. Phys. U.S.S.R.*, **28**, 126 (1955).
- [17] K. R. Atkins, *Proc. Roy. Soc. A* **203**, 240 (1950).
- [18] G. S. Picus, *Phys. Rev.* **94**, 1459 (1954).
- [19] F. D. Manchester, *Can. J. Phys.* **33**, 146 (1955).
- [20] H. Seki, *Phys. Rev.* **128**, 502 (1962).
- [21] F. I. Glick & J. H. Werntz, Jr., *Phys. Rev.* **178**, 214 (1969).
- [22] E. F. Hammel, W. E. Keller & R. H. Sherman, *Phys. Rev. Lett.* **24**, 712 (1970).
- [23] J. S. Brooks & R. B. Hallock, *Phys. Lett. A* **63** 319 (1977).
- [24] J. P. Turneaure & I. Weissman, *J. App. Phys.* **39** 4417 (1968).
- [25] M. Tinkham, *Introduction to superconductivity*, Dover, New York (1996).

- [26] O. Harling , Phys. Rev. A **3**, 1073 (1971).
- [27] L. J. Rodriguez, H. A. Gersch & H. A. Mook, Phys. Rev. A **9**, 2085 (1974).
- [28] E. C. Svensson, V. F. Sears & A. Griffin, Phys. Rev. B **23**, 4493 (1981).
- [29] E. C. Svensson & V. F. Sears, Physica **137B**, 126 (1986).
- [30] R. D. Puff & J. S. Tenn, Phys. Rev. A **1**, 125 (1970).
- [31] G. D. Mahan, Many-Particle Physics, third edition, chapter 11, Kluwer Academic/Plenum Publishers, New York (2000).
- [32] Y. Yu, Ann. Phys. **323**, 2367 (2008), and references therein.
- [33] Y. Yu, Mod. Phys. Lett. **B 29**, 1550068-1 (2015).
- [34] J. D. Reppy & D. Depatie, Phys. Rev. Lett., **12**, 187-189 (1964).
- [35] A. DeMann, S. Mueller & S. B. Field, Cryogenics, **73**, 60 (2016).
- [36] <https://www.temati-uk.com/>.
- [37] The average transfer rate of superfluid  $^4\text{He}$  from pot  $A$  to pot  $B$  is 3 to 4 cubic centimeters per hour, and the corresponding flow velocities in superleaks are the order of a few millimeter per second. The associated kinetic energy ( if applicable) is around  $10^{-29}$  Joule per a  $^4\text{He}$  atom, which corresponds to a temperature scale of  $1\ \mu\text{K}$  (negligible for a helium system at a temperature above  $1.6\ \text{K}$ ). Also, as argued later in the manuscript, the inlet superflow (from the viewpoint of cell  $C$  ) has a velocity smaller than the outlet superflow, so the change of kinetic energy (if applicable) of superflows shall lead to a tiny cooling of cell  $C$  rather than heating it.